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ARE MORE ALTERNATIVES BETTER FOR DECISION-MAKERS? A NOTE ON THE ROLE OF DECISION COST

ABSTRACT. While the traditional economic wisdom believes that an individual will become better off by being given a larger opportunity set to choose from, in this paper we question this belief and build a formal theoretical model that introduces decision costs into the rational decision process. We show, under some reasonable conditions, that a larger feasible set may actually lower an individual's level of satisfaction. This provides a solid economic underpinning for the Simon prediction.

KEY WORDS: bounded rationality, considered subset, decision cost.

JEL CLASSIFICATIONS: D11, D83.

1. INTRODUCTION

The traditional economic wisdom believes that an individual will become better off (or, at least not worse off) by being given a larger opportunity set to choose from. This notion implies that the agent can give up those incremental alternatives that are unwanted without incurring any cost. As pointed out by Conlisk (1988), agents are typically assumed to reason costlessly in regard to a decision about how much information to collect and, given that information, to reason costlessly in relation to an optimal final action. However, it is well recognized pragmatically (see, for instance, Baumol and Quandt, 1964; Williams and Findlay, 1981) that an agent will incur some decision costs when he makes a choice. The experimental economists, such as Smith (1989), Pingle (1992), and Wilcox (1993), use psychological experiments to prove that the decision cost (measured in terms of decision-making time) is often a key factor affecting an agent's choice among alternatives.

Simon (1955) has argued that people may not always follow the rational decision-making rule as described by economists, one of the main reasons for this being the existence of decision costs. Accordingly, he proposes the concept of *bounded rationality* to challenge the traditional economics of the *unbounded* rational decision. In a survey paper, Conlisk (1996) also concludes that the deliberation cost is one of the major reasons why an individual does not act "rationally." However, the decision costs are suppressed in most economic analyses.

The aim of this paper is to shed light on the importance of the decision cost in rational decision theory. While we do not set out to provide yet another proof of the existence of bounded rationality and decision costs, our intention is to set up a formal theoretical model and demonstrate how we should view things differently when having a larger feasible set to choose from, given the existence of decision costs. To be more specific, we will show that, under certain reasonable conditions, the net expected benefit of choosing from a larger feasible set might be smaller. Consequently, the decision-maker might not be better off by being given a larger number of choices. This will provide a solid economic underpinning for the Simon prediction.

This issue has become more important than ever during the present era of information overload. By using an Internet search engine, it is easy for an individual to obtain an enormous feasible set to choose from. People often embrace this increase in the size of the feasible set, which seems to be beneficial at first glance, while failing to realize the implicit costs involved in the decision-making process. When faced with such decision costs, we can observe that in reality some people would actually prefer to choose from a smaller, but more manageable, feasible set. In Sections 2 and 3, we will build a new model that runs counter to the usual model in rational decision theory and use it to express our viewpoint. Section 4 discusses the robustness of our main results and provides numerical calibrations for these results. Section 5 concludes.

2. THE DECISION PROCEDURE AND OPTIMIZATION PROBLEM

Our model shares some concepts with the bounded rationality theory, such as in Simon (1955) and Lipman (1991), and introduces decision costs into the decision-making process. Naturally, there are N bundles in the feasible set that can be ranked according to the order of preferences $x_1 \succ x_2 \succ \cdots \succ x_N$. The same can be done for the corresponding benefits $b(x)$, that is, $b(x_1) > b(x_2) > \cdots > b(x_N)$. Following Simon (1955), we consider

ASSUMPTION 1. A decision-maker (henceforth DM), facing non-trivial decision costs, follows a two-stage decision process as follows:

- Stage 1. (*screening process*): The DM pre-selects those alternative bundles x that fit into the profile of having the potential to become the "best bundle". Specifically, the DM will pick up n bundles from the feasible set that has N alternative bundles. The n selected bundles constitute the *considered subset*.[1]
- Stage 2. (*evaluation process*): The DM evaluates all of the selected bundles x, calculates their corresponding benefits $b(x)$, and chooses the best bundle x_k (and hence derives benefit $b(x_k)$) from the n considered bundles.[2] In addition, the evaluation process entails some evaluation (or decision) costs C,[3] which could be thought of as either a *pecuniary cost* of evaluating or a *psychological disutility* stemming from deliberating over a decision, such as hesitation and uneasiness. For simplicity, the evaluation cost is specified as a linear increasing function with respect to n, i.e. $C(n) = c \cdot n$.

To be more specific, given N alternatives, $\{x_i\}_{i=1}^{N}$, the DM knows there are N possible values of benefits, $\{b_i\}_{i=1}^{N}$.[4] Although the DM may know the possible values of b, he cannot match the correct value b with an alternative x without further evaluation. In other words, the DM does not have exact knowledge regarding the true ranking for N bundles during the screening process; the evaluation process, however, can match the bundles x with their correct benefits b.

By means of both the screening process and the evaluation process, the DM maximizes his net expected benefit, V, as follows:

ASSUMPTION 2. $V \equiv EB(x, n; N) - c \cdot n$.

Because the agent does not have exact knowledge regarding the ranking of x and $b(x)$ for N bundles during the screening process, he can only *ex-ante* consider the following factor, namely, the expected benefit $EB(x, n; N)$. The expected benefit is related to the size of the considered subset n, the benefit arising from the best bundle $b(x_k)$ among the n considered bundles after the evaluation is performed, and the number of alternative bundles in the feasible set N. Notice that, even after evaluation, the DM still cannot be sure that x_k is the best bundle out of all of the alternatives, i.e. x_1, unless he chooses to evaluate all alternative bundles, i.e. $n = N$.

Backward induction is applied to solve this two-stage optimization decision problem. During the second stage, given that n is chosen during the first stage, the DM evaluates these n considered bundles, finds the best optimal bundle x_k (and hence derives benefit $b(x_k)$), and incurs the decision cost $c \cdot n$. By internalizing these possible results, the DM's goal during the first stage is to choose an optimal n so as to maximize V. If the agent chooses a larger n, on the one hand, he has a higher probability of obtaining a higher level of $b(x_k)$ (i.e. a smaller k), but, on the other hand, he will also incur a higher decision cost. If $n = N$, then the agent can obtain the highest benefit $b(x_1)$ for sure, but he will also incur the highest decision cost as measured by $c \cdot N$.

Given the evaluated result $b(x_k)$ and the decision costs $c \cdot n$, the optimal size is:

$$
\begin{aligned}
n^* &= \arg\max_n \quad V \equiv EB(x, n; N) - C \\
&= \sum_{k=1}^{N-n+1} P_k(n; N) \cdot b(x_k) - c \cdot n,
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
P_k(n; N) &= \binom{N-k}{n-1} \Big/ \binom{N}{n} \\
&= (N-k)!/\big((n-1)!(N-n-k+1)!\big) \\
&\quad \cdot (n!\,(N-n)!/N!), \quad 1 \le k \le N-n+1.
\end{aligned}
$$

To obtain the expected benefit $EB(x, n; N)$, we should first calculate the probability of obtaining the bundle x_k, namely $P_k(n; N)$.[5] Given that the sizes of the considered subset and the feasible set are n and N, respectively, the total number of possible combinations of choosing n from N is $\binom{N}{n}$. Furthermore, the number of possible combinations of picking the best bundle x_k out of n considered bundles is $\binom{N-k}{n-1}$. Thus, the probability of obtaining the bundle x_k, $P_k(n; N)$, is the ratio of $\binom{N-k}{n-1}$ to $\binom{N}{n}$ as demonstrated in (1). In an extreme case where $n = N$, we then have $P_1(N; N) = 1$.

A numerical example might be helpful in understanding the inference of $P_k(n; N)$. We set $N = 10$, $n = 3$, and the considered bundles as $(x_i,\ x_j,\ x_k)$, where $i \neq j \neq k$ and $\forall i,\ j,\ k = 1, \ldots, 10$. Accordingly, the total number of possible combinations for the considered subset is $\binom{10}{3} = 10!/(3!\ 7!) = 120$. If the considered subset consists of x_1, i.e. $k = 1$ (of course, x_1 is the best among the three considered bundles), then the considered bundles are $(x_1,\ x_i,\ x_j)$, where $i \neq j$ and $\forall i,\ j = 2, \ldots, 10$. Given that the position of x_1 is certain, obtaining the possible combinations of $(x_1,\ x_i,\ x_j)$ involves choosing two bundles x_i and x_j from the nine bundles x_2–x_{10}. In other words, the number of possible combinations of picking up x_1 from the three considered bundles is $\binom{9}{2} = 9!/(2!\ 7!) = 36$. Thus the probability of obtaining the bundle x_1 is $P_1(n = 3;\ N = 10) = 36/120 = 3/10$. If $k = 2$ and x_2 (the second best among all bundles) is the best among three considered bundles, then the considered subset must rule out x_1. Hence the number of possible combinations that would pick up x_2 is $\binom{8}{2} = 8!/(2!\,6!) = 28$. Accordingly, we have $P_2(n = 3; N = 10) = 28/120 = 7/30$. We can infer the probability of obtaining x_k as $P_k(n = 3; N = 10) = \binom{10-k}{2}/120$ in the case where $N = 10$ and $n = 3$. By the same logic, in the general case the probability of obtaining x_k is $P_k(n; N) = \binom{N-k}{n-1}\Big/\binom{N}{n}$, as expressed in (1).

Define a cumulative probability function as

$$\Phi_k(n; N) \equiv \Pr(b(x) \geqslant b(x_k)) = \sum_{i=1}^{k} P_i(n; N),$$

and we have:

LEMMA 1. $\Phi_k(n+1; N) \geqslant \Phi_k(n; N)\ \ \forall k.$

Proof. See Appendix A.

Furthermore, according to the concept of "stochastic dominance" defined by Hardar and Russell (1969), the following lemma is also obtained immediately:

LEMMA 2. *Suppose that there are two probability density functions $f(\cdot)$ and $g(\cdot)$ with respect to a random variable x. Let us denote their corresponding cumulative probability density functions as $F(x)$ and $G(x)$, respectively. If $F(x) \geq G(x)\ \ \forall x$ and $b(\cdot)$ is a decreasing function of x, then $\sum b(x) \cdot f(x) \geqslant \sum b(x) \cdot g(x)$ must hold.*

From the relationship $b(x_1) > b(x_2) > \cdots > b(x_N)$ and Lemmas 1 and 2, we establish:

PROPOSITION 1. *Under Assumptions 1 and 2, then $EB(x, n+1; N) \geqslant EB(x, n; N)$.*

This indicates that the expected benefit EB increases with the size of the considered subset n.

According to Proposition 1, the marginal decision-making approach indicates that the optimal size of the considered subset n^* satisfies the condition in which the marginal benefit of changing n (MB) equals its marginal cost (MC). That is,

$$MB(n^*; N) \equiv EB(x, n^*; N) - EB(x, n^* - 1; N) = c. \qquad (2)$$

One point should be noted. Since n is a discrete variable, (2) may not always be satisfied. If so, the optimal n^* is an integer such that

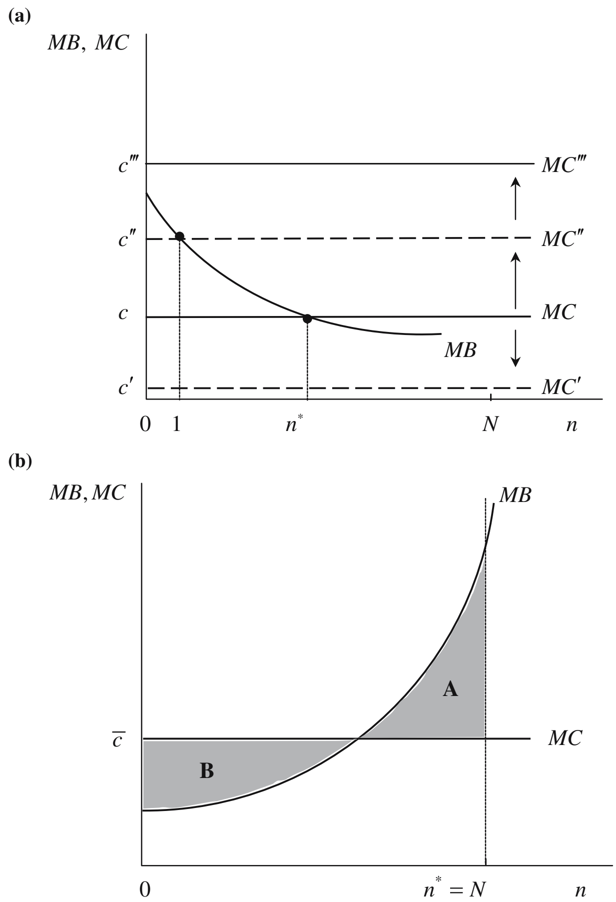

Figure 1.

$$EB(x, n^*; N) - EB(x, n^* - 1; N) > c \quad \text{and}$$
$$EB(x, n^* + 1; N) - EB(x, n^*; N) < c. \tag{2'}$$

However, in order to make our analysis more treatable and without significant loss of generality, we focus on the case where (2) is held.

To be sure that the optimal n^* is acceptable to the DM, the following condition.

ASSUMPTION 3. $V^* = EB(x, n^*; N) - c \cdot n^* \geqslant 0$ is required, implying that, based on the optimal size of the considered subset, the net expected benefit is non-positive.

We now explore the role of evaluation costs in determining the optimal size of the considered subset in Figure 1(a) and (b). In Figure 1(a) and (b), the MC curve is horizontal due to the assumption of a fixed marginal decision cost. Since MB can be either negatively or positively related to n, we need to discuss these two possible cases. We start the discussion with the case where MB has negative relationship with n, as described in Figure 1(a). In the case of the traditional theory of rational choice, the decision cost is trivial and infinitesimal (hence, the marginal decision cost is close to zero, say c'). Thus, we yield a corner solution, which indicates that the optimal size of the considered subset is that of the feasible set, i.e. $n^* = N$. However, in most cases, the cost of evaluation is non-trivial, namely, the marginal decision cost is c. In such a case, there is an interior solution and the intersection of the curves MB and MC determines the optimal size of the considered subset n^*. If the marginal decision cost is increased from c to c'', as depicted in Figure 1(a), the optimal number of the considered bundles is unity, because MB intersects MC at 1. This potentially implies that the DM may randomly select his alternative and the *ex post* benefit of his decision will be completely dependent upon luck. An extreme case is that, if the marginal cost is extremely high, say c''', the optimal set of the considered bundles will be empty. In such a situation, the DM will give up all possible alternatives. These results show that the optimal size of the considered subset n^* may fall short of the total number of alternatives N.

We turn to the case where MB is positively related to n. Let $\bar{c}$ be a critical level of marginal cost such that the net expected benefit is zero, i.e. $EB = C$, which implies that the area A is equal to the area B, as illustrated in Figure 1(b). If $MC \leq \bar{c}$, the optimal n^* will entail a positive net expected benefit V (due to the area A being larger than the area B). Because the net expected benefit increases with n, the DM will increase the number of considered bundles until $n^* = N$. On the contrary, if $MC > \bar{c}$, the DM will decrease the size of n as much as he possibly can due to $EB < C\ \forall n$. In this case, $n^* = 0$ may be a possible solution. In other words, given that MB is upward sloping, the optimal number of considered bundles is a corner solution, i.e. either $n^* = 0$ or $n^* = N$. We summarize the above results as:

PROPOSITION 2. *Under Assumptions 1–3, in the presence of non-trivial decision costs, the DM will not necessarily consider all alternatives in the feasible set.*

3. ARE MORE ALTERNATIVES BETTER FOR THE DM?

To shed light on our main point, we will henceforth place our focus on the case of interior solutions (hence the case where MB is decreasing with n). Now, we consider a larger feasible set, say, where the size of the feasible set increases from N to $N+1$. Corresponding to this size of feasible set of $N+1$, the probability of picking x_k from n considered bundles is denoted by $P_k(n; N+1)$, which is expressed as:

$$\begin{aligned}
&P_k(n; N+1)\\
&\quad = \binom{N-k+1}{n-1} \Big/ \binom{N+1}{n}\\
&\quad = \frac{(N-k+1)!}{(n-1)!\,(N-n-k+2)!} \cdot \frac{n!\,(N-n+1)!}{(N+1)!}\\
&\quad = \begin{cases} \frac{(N-n+1)(N-k+1)}{(N+1)(N-n-k+2)} \cdot P_k(n; N) & \text{if } 1 \leqslant k \leqslant N-n+1,\\ \frac{(N-n+1)!}{(N+1)!} & \text{if } k = N-n+2,\\ 0 & \text{if } k > N-n+2. \end{cases}
\end{aligned}$$

According to (3), we have

LEMMA 3. $\Phi_k(n; N) \geqslant \Phi_k(n; N+1) \ \ \forall k$.

Proof. See Appendix B.

After the size of the feasible set is increased by 1, the rank and the corresponding benefits are specified as $\tilde{x}_1 \succ \tilde{x}_2 \succ \cdots \succ \tilde{x}_N \succ \tilde{x}_{N+1}$ and as $b(\tilde{x}_1) > b(\tilde{x}_2) > \cdots > b(\tilde{x}_N) > b(\tilde{x}_{N+1})$, respectively. Accordingly, from Lemmas 2 and 3, it is easy to derive the difference between $EB(x,\ n; N+1)$ and $EB(x,\ n; N)$ as:

$$\begin{aligned} &EB(x, n; N+1) - EB(x, n; N) \\ &\quad = \sum_{k=1}^{N-n+2} [P_k(n; N+1) - P_k(n; N)] \cdot b(\tilde{x}_k) \\ &\qquad + \sum_{k=1}^{N-n+2} P_k(n; N) \cdot [b(\tilde{x}_k) - b(x_k)]. \end{aligned} \tag{4}$$

In order to make our point more striking, we further assume that the DM does not care about the intrinsic benefit of the chosen bundle, but simply about the rank of the chosen bundle among the feasible set. Specifically, we propose:

ASSUMPTION 4. $b(x_k) = \phi(k)$, where $\phi' < 0$.

The DM does not know the actual benefit arising from any alternative bundle before the evaluation process is undertaken, and therefore his goal *may* be to maximize satisfaction brought in by the optimal bundle according to its "rank". The benefit function may be specified more explicitly as $b(x_k) = z - k^{\gamma}$, where z and γ are coefficients. To examine the validity of the proposition, in Section 4, based on this specification, we will perform numerical simulations.

If the DM is only concerned with whether the selected bundle is the best one out of the considered subset based on its "rank" within the feasible set, and is not concerned with whether the best bundle is chosen from either N alternative bundles or $(N+1)$ bundles, the satisfaction (the *feeling*) from obtaining the highest ranking bundle from a considered subset out of the feasible set $(N+1)$ will be the same as getting it out of the feasible set N.[6] That is, $b(\tilde{x}_k) = b(x_k)$, $k = 1, 2, \ldots, N$ and $b(x_N) > b(\tilde{x}_{N+1})$. Given this assumption, we modify (4) as:

$$EB(x, n; N+1) - EB(x, n; N) = \sum_{k=1}^{N-n+2} [P_k(n; N+1) - P_k(n; N)] \cdot b(x_k). \tag{5}$$

Proposition 3 is immediately derived from (5).

PROPOSITION 3. *Under Assumptions* 1, 2 and 4, $EB(x, n; N) \geqslant EB(x, n; N+1)$ $\forall n$ and N.

Proof. See Appendix C.

Proposition 3 indicates that, a larger size of feasible set may result in a lower expected benefits EB given a particular size of the considered subset. However, more alternatives may lead the DM to change his decision concerning the size of the considered subset's optimal n, and, as a result, alter his expected benefits EB and decision costs C. Therefore, in what follows, we will further explore the net change in welfare of the DM.

It easily follows from (2) that an increase in the number of alternatives from N to $N+1$ has an ambiguous effect on the optimal n^*, depending on the relative curvature of $EB(x, n; N)$ and $EB(x, n; N+1)$. Based on this and Proposition 3, we then have:

PROPOSITION 4. *Under Assumptions* 1–4*, a larger feasible set may make the DM worse off.*

Proof. See Appendix D.

A remark should be made here: To make our argument more striking, Proposition 4 is established under Assumption 4. However, we do not intend to claim that more alternatives should make the DM worse off, particularly when Assumption 4 is relaxed. Now we turn to the discussion concerning the robustness of Proposition 4.

4. DISCUSSION AND NUMERICAL SIMULATIONS

Propositions 3 and 4 are established under Assumption 4 in which the DM does not care about the intrinsic benefit of the chosen bundle, but simply about the rank of the chosen bundle among the feasible set. However, in some cases the DM does care about the intrinsic benefit of the alternative bundle, for instance, utility from consumption. One may thus inquire about the robustness of these propositions.

In what follows, we will show that Propositions 3 and 4 may still hold, even though the DM is concerned with not only the "rank" of the chosen bundle, but also the "level" of benefit derived from a chosen bundle. Suppose that the additional

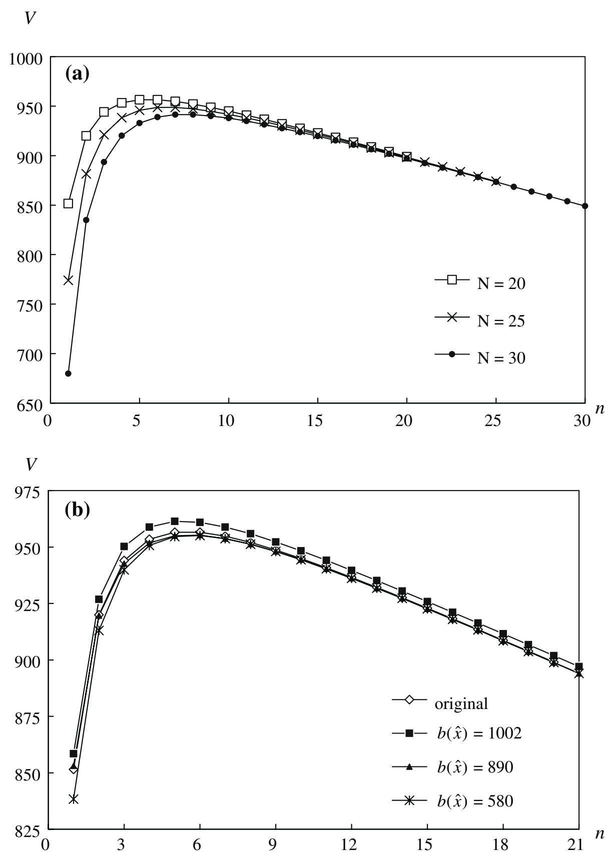

Figure 2.

alternative is $\hat{x}$, when the size of the feasible set increases from N to $N+1$. By letting $x_1 \succ \cdots \succ x_{m-1} \succ \hat{x} \succ x_m \succ \cdots \succ x_N$, we have

$$b(\tilde{x}_k) = b(x_k) \text{ for } k = 1, \ldots, m-1$$
$$\text{and } b(\tilde{x}_k) > b(x_k) \text{ for } k = m, \ldots, N.$$

Accordingly, the second item in (4) is positive and decreasing with m. For example, in extreme cases, if $\hat{x} \succ x_1$, this will result in the largest value for the second item in (4) and if $x_N \succ \hat{x}$, this will result in the smallest value for the second item in (4). Given that the sign of the first item in (4) must be negative (inferred by Lemma 3), Proposition 3 is more likely to be true when m is larger (or $b(\hat{x})$ is smaller). Provided that the negative effect of the first term in (4) is substantially strong, Propositions 3 and 4 may be valid.

We next perform two sets of numerical simulations in order to explicitly show the possibility of Proposition 4.

(1) *The DM is only concerned with the rank of a chosen bundle*

Let $b(x_k) = 1000 - k^2$ and $C = c \cdot n = 5\,n$, where $k = 1, \ldots, N$. Consider three possible sizes of the feasible set, say 20, 25, and 30, respectively. Figure 2(a) shows that, given that N is 20, 25 and 30, the associated optimal sizes of considered subset n^* are 5 (or 6), 7 and 8, respectively. Furthermore, we can evaluate their corresponding net expected benefits V as 956.5, 948.75 and 941.4. Obviously, the largest net expected benefit (956.5) is derived from the smallest feasible set, $N = 20$, rather than the larger feasible sets, $N = 25$ or $N = 30$.

(2) *The DM cares not only about the rank of a chosen bundle but also its intrinsic value*

Consider a benefit set $\{b(x_k)\}_{k=1}^{20}$, namely F_1, consisting of {999, 996, 991, 984, 975, 964, 951, 936, 919, 900, 879, 856, 831, 804, 775, 744, 711, 676, 639, 600}. The setting of the decision cost is the same as in (1). Obviously, the benefit pertaining to the best bundle is 999 while the benefit from the worst bundle is 600. Now, assume that there is an additional bundle $\hat{x}$ and that its corresponding benefit is denoted by $b(\hat{x})$. The benefit set with a larger number of alternatives, $N = 21$ (including $b(\hat{x})$), is defined as F_2.

We consider three possible cases: (i) $b(\hat{x})$ is greater than all of elements of the set F_1 and is specified as 1,002 ($>$ 999); (ii) $b(\hat{x})$ is smaller than all of elements of the set F_1 and is specified as 580 ($<$600); (iii) $b(\hat{x})$ is a middle value among the elements of the set F_1 and is specified as 890. Given these specifications, Figure 2(b) shows that, under the feasible set F_1, the optimal size of the considered subset is $n^* = 5$ or 6, (and hence the maximum net expected benefit is $V^* = 956.5$); however, in response to an increase in the feasible set (i.e. under the larger set F_2), the optimal size of the considered subset may either increase ($n^* = 6$ in case (ii) and (iii)) or decrease ($n^* = 5$ in case (i)). The result is as predicted by our deductions in Section 3.

Of great importance, when we consider a larger feasible set F_2, as indicated by Figure 2(b), is that the maximum net expected benefits become 961.38 in case (i), 955.22 in case (ii), and 955.07 in case (iii), respectively. Given that the maximum net expected benefit is $V^* = 956.5$ under F_1 with $N = 20$, except in case (i), the net expected benefit V does not increase with the larger size $N = 21$ of the feasible set F_2. In other words, more alternatives do not necessarily make the DM better off unless they can provide better choices for the DM.

5. CONCLUDING REMARKS

Conlisk (1996, p. 671) stressed that, for an individual with bounded rationality, heuristics often provide adequate solutions that are cheap, whereas more elaborate approaches would be unduly expensive. By taking decision costs into account, we set up a formal model and show that having more alternatives may not make a DM better off. The numerical simulations that we perform also support this argument. This result, that runs contrary to the traditional rational choice theory, has important implications for DM.

APPENDIX A

The Proof of Lemma 1. Given the n considered bundles, the definition of Φ_k immediately yields:

$$\Phi_k(n;N) = \sum_{i=1}^{k} P_i(n;N) = \begin{cases} \sum_{i=1}^{k} \frac{(N-i)!}{(n-1)!\,(N-n-i+1)!} \cdot \frac{n!\,(N-n)!}{N!} & \text{if } k \le N-n+1, \\ 1 & \text{if } k > N-n+1. \end{cases} \tag{A1}$$

In the case of a larger considered subset with $(n+1)$ bundles, the cumulative probability function will be changed into

$$\Phi_k(n+1;N) = \begin{cases} \sum_{i=1}^{k} \frac{(N-i)!}{n!\,(N-n-i)!} \cdot \frac{(n+1)!\,(N-n-1)!}{N!} & \text{if } k \le N-n, \\ 1 & \text{if } k > N-n. \end{cases} \tag{A2}$$

By rearrangement, $\Phi_k(n+1;N)$ can be written as

$$\Phi_k(n+1;N) = \left(\frac{n+1}{n}\right) \cdot \sum_{i=1}^{k} P_i(n;N) \cdot \left(\frac{N-n-i+1}{N-n}\right) = \left(\frac{n+1}{n}\right) \cdot \sum_{i=1}^{k} P_i(n;N) \cdot \left(1 - \frac{i-1}{N-n}\right). \tag{A3}$$

According to (A3), we further derive

$$\Phi_{k+1}(n+1;N) = \left(\frac{n+1}{n}\right) \cdot \sum_{i=1}^{k+1} P_i(n;N) \cdot \left(1 - \frac{i-1}{N-n}\right) = \sum_{i=1}^{k+1} P_i(n;N) + \frac{1}{n} \cdot \sum_{i=1}^{k+1} P_i(n;N) - \left(\frac{n+1}{n}\right) \cdot \sum_{i=1}^{k+1} P_i(n;N) \cdot \left(\frac{i-1}{N-n}\right)$$

$$= \Phi_{k+1}(n;N) + \frac{1}{n}\cdot\sum_{i=1}^{k} P_i(n;N)$$
$$- \left(\frac{n+1}{n}\right)\cdot\sum_{i=1}^{k} P_i(n;N)\cdot\left(\frac{i-1}{N-n}\right)$$
$$+ \frac{1}{n}\cdot P_{k+1}(n;N)\cdot\left[1-\frac{(n+1)k}{N-n}\right].$$

Let $\Delta_n\Phi_k \equiv \Phi_k(n+1;N) - \Phi_k(n;N)$ and $P_i(n;N) = P_i$, then the above equation can be rewritten as

$$\Phi_{k+1}(n+1;N) = \Phi_{k+1}(n;N) + \Delta_n\Phi_k + \frac{1}{n}\cdot P_{k+1}\cdot\left[1-\frac{(n+1)k}{N-n}\right]$$
$$\Rightarrow \Delta_n\Phi_{k+1} = \Delta_n\Phi_k + \Theta_{k+1}, \qquad \text{(A4)}$$

where

$$\Theta_{k+1} \equiv \frac{1}{n}\cdot P_{k+1}\cdot\left[1-\frac{(n+1)k}{N-n}\right].$$

Suppose that there exists a critical k^* such that $k^* = (N-n)/(n+1)$. When $k < k^*$, then $[1-(n+1)k/(N-n)] > 0$; otherwise, when $k > k^*$, then $[1-(n+1)k/(N-n)] < 0$. Since P_{k+1} and $[1-(n+1)k/(N-n)]$ both decrease with k, Θ_{k+1} is also a decreasing function of k. Accordingly, Figure A.1 follows equation (A4) and depicts the

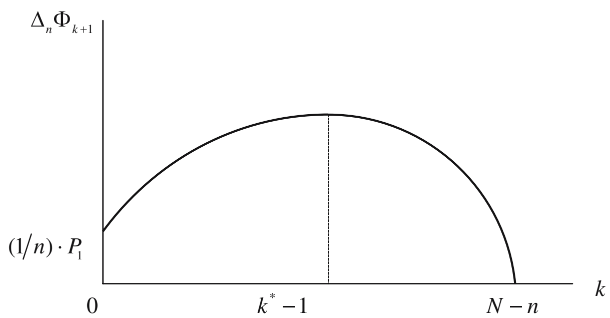

Figure A.1.

relationship between k and $\Delta_n\Phi_{k+1}$, which indicates that $\Theta_{k+1} > 0$ (i.e. $\Delta_n\Phi_{k+1} > \Delta_n\Phi_k$) if $k < k^*$ and $\Theta_{k+1} < 0$ (i.e. $\Delta_n\Phi_{k+1} < \Delta_n\Phi_k$) if $k > k^*$. In Figure A.1, in order to sketch the relationship between k and $\Delta_n\Phi_{k+1}$, we have utilized the following relationships $\Phi_1(n+1;N) = (n+1)/n \cdot P_1 > P_1 = \Phi_1(n;N)$, $\Delta_n\Phi_1 = (1/n)\cdot P_1$, and $\Phi_{N-n+1}(n+1;N) = \Phi_{N-n+1}(n;N)$, i.e. $\Delta_n\Phi_{N-n+1} = 0$. Obviously, Figure A.1 indicates that $\Delta_n\Phi_k \geqslant 0 \;\; \forall k$, implying that $\Phi_k(n+1;N) \geqslant \Phi_k(n;N) \;\; \forall k$. □

APPENDIX B

The Proof of Lemma 3. According to (3), we have

$$\begin{aligned}\Phi_k(n;N+1) &= \sum_{i=1}^{k} P_i(n;N+1)\\ &= \sum_{i=1}^{k} P_i(n;N)\cdot\left(\frac{N-n+1}{N+1}\right)\cdot\left(\frac{N-i+1}{N-n-i+2}\right)\\ &= \Phi_k(n;N) + \sum_{i=1}^{k} P_i(n;N)\\ &\quad\times\left[\left(\frac{N-n+1}{N+1}\right)\cdot\left(\frac{N-i+1}{N-n-i+2}\right)-1\right].\end{aligned}$$

Letting $\Delta_N\Phi_k = \Phi_k(n;N+1) - \Phi_k(n;N)$, then

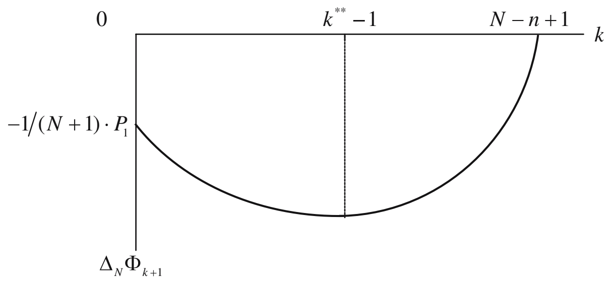

Figure A.2.

$$\Delta_N \Phi_k = \sum_{i=1}^{k} P_i(n;N) \cdot \times \left[\left(\frac{N-n+1}{N+1} \right) \cdot \left(\frac{N-i+1}{N-n-i+2} \right) - 1 \right]$$

and

$$\Delta_N \Phi_{k+1} = \Delta_N \Phi_k + P_{k+1}(n;N) \times \left[\left(\frac{N-n+1}{N+1} \right) \cdot \left(\frac{N-k}{N-n-k+1} \right) - 1 \right]. \tag{A5}$$

In Equation (A5) there exists a critical $k^{**} = (N-n+1)/n$ such that $\Delta_N \Phi_{k+1} = \Delta_N \Phi_k$, and $\Delta_N \Phi_{k+1} < \Delta_N \Phi_k$ if $k < k^{**}$, while $\Delta_N \Phi_{k+1} > \Delta_N \Phi_k$ if $k > k^{**}$. It follows from equation (3) that $\Delta_N \Phi_1 = P_1(n; N+1) - P_1(n;N) < 0$ if $k = 1$, and $\Delta_N \Phi_k = 0$ if $k \geqslant N-n+2$. Based on these inferences, we sketch the relationship between k and $\Delta_n \Phi_{k+1}$ in Figure A.2. Figure A.2 indicates that $\Delta_N \Phi_k \leq 0 \ \forall k$, i.e. $\Phi_k(n;N) \geqslant \Phi_k(n;N+1) \ \forall k$. □

APPENDIX C

The Proof of Proposition 3. From Lemmas 2 and 3, it is clear that $EB(x,n;N+1)$ will be less than $EB(x,n;N)$ for all n if the DM is only concerned with the rank of a chosen bundle. □

APPENDIX D

The Proof of Proposition 4. Assume that the optimal size of the considered subset is n^* when the size of a feasible set is N, and that the optimal size of the considered subset is n^{**} when the size of a feasible set is $N+1$. Their corresponding optimal net expected benefits are V^* and V^{**}. Accordingly, by defining $\Delta n \equiv n^* - n^{**}$, we have

$$\begin{aligned} V^* - V^{**} &= [EB(x, n^*; N) - c \cdot n^*] \\ &\quad - [EB(x, n^{**}; N+1) - c \cdot n^{**}] \\ &= [EB(x, n^{**}; N) - EB(x, n^{**}; N+1)] \\ &\quad + [EB(x, n^*; N) - EB(x, n^* - \Delta n; N) - c \cdot \Delta n] \\ &\equiv \Omega_1 + \Omega_2. \end{aligned} \tag{A6}$$

In (A6) the term Ω_1 is positive due to $EB(x, n^{**}; N) > EB(x, n^{**}; N+1)$ based on Proposition 3. Moreover, if $n^{**} = n^*$, the term Ω_2 will be reduced to zero. Thus, $V^{**} < V^*$ is true and Proposition 4 holds.

If $n^{**} \neq n^*$, more discussions are needed. If $\Delta n = 1$, Ω_2 is zero since the optimal condition (2) indicates that $MB(n^*; N) \equiv EB(x, n^*; N) - EB(x, n^* - 1; N) = c$. Given Proposition 3, the result $V^{**} < V^*$ is still valid in such a case.

If $\Delta n \neq 1$, for example, $|\Delta n| = 2$, then

$$\begin{aligned} \Omega_2 &= EB(n^*; N) - EB(n^* - 2; N) - c \cdot 2 \\ &= [EB(n^*; N) - EB(n^* - 1; N)] \\ &\quad + [EB(n^* - 1; N) - EB(n^* - 2; N)] - 2c \\ &\equiv MB(n^*; N) + MB(n^* - 1; N) - 2c, \end{aligned} \tag{A7}$$

when $\Delta n = 2$, and

$$\begin{aligned} \Omega_2 &= EB(n^*; N) - EB(n^* + 2; N) - c \cdot (-2) \\ &= [EB(n^*; N) - EB(n^* + 1; N)] \\ &\quad + [EB(n^* + 1; N) - EB(n^* + 2; N)] + 2c \\ &\equiv -[MB(n^* + 1; N) + MB(n^* + 2; N)] + 2c, \end{aligned} \tag{A7$'$}$$

when $\Delta n = -2$. With an interior solution, the optimal conditions (2) or (2′) are satisfied, implying that

$$MB(n; N) \equiv EB(n; N) - EB(n - 1; N) \stackrel{>}{<} c \text{ if } n \stackrel{<}{>} n^*. \tag{A8}$$

Therefore, we can conclude

$$\begin{aligned} \Omega_2 &= MB(n^*; N) + MB(n^* - 1; N) - 2c > c + c - 2c \\ &= 0 \text{ if } \Delta n = 2, \end{aligned}$$

$$\Omega_2 = -[MB(n^* + 1; N) + MB(n^* + 2; N)] + 2c > -c - c + 2c = 0 \quad \text{if } \Delta n = -2.$$

That is to say, the term Ω_2 of (A6) is always positive whenever $\Delta n = 2$ or -2. Based on the similar inference, the result can be applied to Δn being any integer number. Moreover, in the extreme case where $n^* = N$ and $n^{**} = N + 1$, $V^{**} < V^*$ is also true since $b(x_1) - c \cdot N > b(x_1) - c \cdot (N + 1)$. Thus, Proposition 4 is proved. □

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NOTES

1. The term "considered subset" is taken from Simon (1955).
2. For example, if $k = 3$, this means that x_3 (the third best among all) is the best among the n considered bundles that are picked up during the screening process.
3. Takahashi and Takayanagi (1985) propose two other approaches to search for the optimal bundle. One is the "fixed-size procedure" and the other is the "sequential procedure." Under the fixed-size procedure, the decision-maker comprehensively surveys all possible alternatives before the evaluation process begins. On the other hand, when the "sequential procedure" is adopted, the DM considers one alternative at a time and accepts such an alternative when it reaches the acceptable level; otherwise, he rejects it and returns to the screening process. Obviously, these two different procedures give rise to different decision costs.
4. In this paper x can be regarded as either a consumption bundle for a consumer, or an investment project for an entrepreneur. Thus, $b(\cdot)$ is regarded as the utility function or return function, respectively. When making an investment decision, the entrepreneur's return must involve some uncertainty. Therefore, the DM may only have *a priori* information that each alternative x has a benefit b drawn from some distribution, or,

more specifically, *a priori* benefit of x is i.i.d. such that $b(x) \sim F(b)$. In such a case, a Bayesian decision procedure will be applied. To keep matters simple and focus on our main point, we do not explicitly deal with this problem. Nevertheless, we can think of $b(\cdot)$ as the expected benefit of alternative x in order to simplify our model. We are grateful to an anonymous referee for bringing this point to our attention.

5. Notice that since the DM picks up the n considered bundles randomly from the feasible set, he cannot therefore know the exact value of x_k.
6. For example, the DM has the same satisfaction level if he can obtain the first-best bundle (i.e. x_1 or $\tilde{x}_1$) regardless of whether it is chosen from the N or $(N+1)$ bundles.

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